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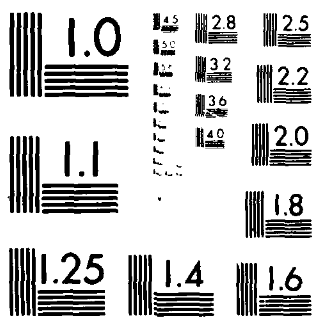

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SENSITIVITY AND PARAMETRIC BOUND ANALYSIS OF AN ELECTRIC POWER GENERATION GP MODEL: OPTIMAL STEAM TURBINE EXHAUST ANNULUS AND CONDENSER SIZES

by

A. V. Fiacco
A. Ghaemi

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The George Washington University
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PARAMETRIC ANALYSIS	STEAM TURBINE EXHAUST ANNULUS SIZE	

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As a further demonstration of practical applicability, a posynomial programming model for the optimal selection of steam turbine exhaust annulus and condenser sizes developed by Ecker and Wiebking is solved and analyzed for optimal value function sensitivities and parametric optimal value bounds. This provides the essential information for conducting a thorough cost benefit analysis over the large parameter changes considered, for this extremely important class of energy applications.

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1. Introduction

Currently, about 25 percent of the energy consumed in the United States is in the form of electric energy generated by power plants. More than 80 percent of this electricity production is due to steam power plants [15]. This reveals the important role of steam turbine power plants as a major contributor to energy production and suggests the efficient design and operations of these plants as a vitally important objective in this era of energy shortage.

The efficient design of steam power plants has been a source of contention since the late 19th century. During the past 80 years, the efficiency of the steam power plants has been increased from about 4 percent to about 33 percent. That is, only about one-eight as much fossil fuel is now required per unit of electric energy production, compared to that of 1900 [12]. This advance has been due to a variety of reasons, such as mechanical design improvements, advances of thermodynamics, and metallurgical developments that have made high temperature boilers and turbines possible.

There have been numerous studies addressed to the optimal design and economic operation of the various components of steam turbine power plants. A good survey of these studies is given by Wiebking [15]. The major components consist of the boiler, steam turbine, condenser, pump and generator. Figure 1 depicts a simplified layout of these components in a steam turbine power plant.

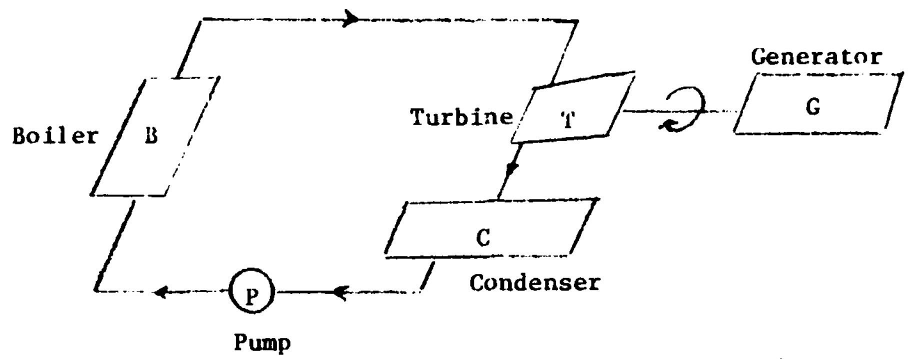

Figure 1. A simplified layout of a steam turbine power plant

It is well known that the optimal design of the subsystems and individual components of a given system do not necessarily lead to an overall optimal design of the given system. In power plants, however, Wilson and Malouf [17] and Wilson [16] have shown that the overall optimal sizing of the condensing system and turbine exhaust annulus can be closely approximated by optimizing these systems independently of the remaining components. Based on this finding Wiebking [15] has formulated and solved numerous geometric programming models with different design configurations, for optimal sizing of the turbine exhaust and condensing systems of steam turbine power plants.

In the present paper, we analyze one of these GP models, studied by Ecker and Wiebking [3], [4], for optimal value function sensitivity and parametric bounds, with respect to many of the model parameters. The sensitivity analysis methodology used here is based on the work of Fiacco [6], and the optimal value bound approach was proposed by Fiacco

and implemented by Fiacco and Ghaemi [10]. Ghaemi has successfully demonstrated the applicability of this bound calculation procedure to several nontrivial problems [11].

It is not the purpose of this study to elaborate on and justify the choice of constraints or objective function, or their derivation, involved in the model under consideration. Readers interested in these aspects may refer to [15]. Here, after presenting the mathematical formulation of the model and identifying the variables and parameters, we report on the solution of a convex equivalent of the model and analyze it for sensitivities and optimal value bounds, using the latest version of our penalty function-sensitivity analysis computer program SENSUMT [10].

The paper is organized as follows. Section 2 briefly reviews the basic idea of the optimal value bound calculation, its implementation on the computer, and its application to GP models. Section 3 lists the model parameters and variables and problem formulation. The usual exponential transformation is applied to convert the GP to a standard convex nonlinear programming (NLP) problem. Sections 4 and 5 present the solution, sensitivity, and bound analysis results.

2. Parametric Optimal Value Bounds

In this section we briefly outline the procedure for calculating piecewise linear parametric upper and lower bounds on the optimal value function of the convex right-hand side perturbation problem, which is known to have a convex optimal value function. Making additional use of some recent developments by Dembo [2], we will demonstrate the applicability of this technique to geometric programs.

Parametric Bounds on the Optimal Value Function of Convex Right-Hand Side Problems CR(ε)

Consider the following parametric programming problem R(ε):

$$\begin{array}{lll} \text{Minimize} & f(x) & \\ \text{subject to} & g_i(x) \geq \varepsilon_i \;, \quad i=1,m & \qquad R(\varepsilon) \\ & h_j(x) = \varepsilon_j \;, \quad j=m+1,p \;, & \end{array}$$

where the decision variable $x \in E^n$, f, g_i, and h_j: $E^n \to E^1$ are C^2 and the parameter vector $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_p)$ is in E^p. If $f(x)$ and $-g_i(x)$, $i=1,m$, are convex and $h_j(x)$, $j=m+1,p$ are affine, then the problem $R(\varepsilon)$ is convex and will be designated as $CR(\varepsilon)$. It is well known that $f^*(\varepsilon)$, the optimal value function of the problem $CR(\varepsilon)$ is convex over any convex subset of the parameter space on which a solution is defined.

The convexity of the optimal value function $f^*(\varepsilon)$ of the problem $CR(\varepsilon)$ allows for a simple calculation of parametric linear upper and lower bounds of this function when any of the problem parameters is perturbed. In our application, this will require the solution and corresponding optimal value function sensitivity information for both perturbed and unperturbed problems. The idea follows immediately from two well known properties of convex functions:

(i) Any line connecting two points on the graph of a convex function does not underestimate that function between the points.

(ii) Any tangent line to the graph of a convex function does not overestimate that function.

These two properties lend themselves in a natural way to the calculation of parametric bounds on the optimal value function of the problem $CR(\varepsilon)$ under large perturbations of any of the problem parameters, say ε_i.

The idea of using properties (i) and (ii) for calculating bounds on the optimal value function of the problem with a convex or concave optimal value function and for estimating bounds on nonconvex problems by way of estimating problems with convex or concave optimal value function is due to Fiacco [7]. Fiacco also notes that techniques for generating

underestimating problems that are jointly convex in the decision variable and the parameter (hence, that have a convex optimal value function [13]) are already available for problems that are jointly separable [5] or jointly factorable [14], thus making it possible to calculate lower bounds using known procedures. The first computational implementation of this approach was developed and reported by Ghaemi [11], who also derived the formulas for calculating a jointly convex over-estimating problem of a factorable program, thus making it possible to numerically implement the proposed scheme for calculating upper bounds when the problem functions are factorable (as they inevitably are, in practice).

Suppose $f^*(\varepsilon)$ is differentiable. (Well-known conditions guaraneeing this were given by Fiacco [6].)

Basis for Calculation of Bounds on $f^*(\varepsilon)$ of the Problem $CR(\varepsilon)$

For simplicity, suppose that some ε_i is perturbed from $\bar{\varepsilon}_{i1}$ to $\bar{\varepsilon}_{i2}$, while the remaining parameters are fixed at their base values.

Step 1. Solve the unperturbed problem and obtain $f^*(\varepsilon_i)$ and $df^*(\varepsilon_i)/d\varepsilon_i$ at $\varepsilon_i = \bar{\varepsilon}_{i1}$. Under general conditions (e.g., see Armacost and Fiacco [1]),

$$\frac{df^*(\varepsilon_i)}{d\varepsilon_i} = \begin{cases} u_i(\varepsilon_i), & i=1,\ldots,m \\ -w_i(\varepsilon_i), & i=m+1,p . \end{cases}$$

where u and w are optimal Lagrange multipliers associated with inequality and equality constraints of $CR(\varepsilon)$, respectively.

Step 2. Resolve the problem and obtain $f^*(\varepsilon_i)$ and $df^*(\varepsilon_i)/d\varepsilon_i$ at $\varepsilon_i = \bar{\varepsilon}_{i2}$.

Step 3. Derive the equation of the line passing through the points $\left(\bar{\varepsilon}_{i1}, f^*(\bar{\varepsilon}_{i1})\right)$ and $\left(\bar{\varepsilon}_{i2}, f^*(\bar{\varepsilon}_{i2})\right)$. This line provides a parametric upper bound $\bar{f}^*(\varepsilon_i)$ for $f^*(\varepsilon_i)$ as a function of $\varepsilon_i \in [\bar{\varepsilon}_{i1}, \bar{\varepsilon}_{i2}]$.

Step 4. Derive the equation of the tangent lines to $f^*(\varepsilon_i)$ at the above two points with the slopes

$$\left.\frac{df^*(\varepsilon_i)}{d\varepsilon_i}\right|_{\varepsilon_i=\bar{\varepsilon}_{i1}} \quad \text{and} \quad \left.\frac{df^*(\varepsilon_i)}{d\varepsilon_i}\right|_{\varepsilon_i=\bar{\varepsilon}_{i2}}$$

calculated in Steps 1 and 2, respectively. The maximum of these two lines provides a piecewise-linear parametric lower bound $\underline{f}^*(\varepsilon_i)$ for $f^*(\varepsilon_i)$ as a function of ε_i in $[\bar{\varepsilon}_{i1}, \bar{\varepsilon}_{i2}]$.

The lines obtained in Steps 3 and 4 provide the desired bounds, forming a triangle which encloses the optimal value function $f^*(\varepsilon_i)$ over the given range of ε_i. Further, a smooth estimate of $f^*(\varepsilon_i)$ over the given interval can also be made by fitting any differentiable convex function that passes through the points given in Step 3, having the corresponding slopes at these points obtained in Steps 1 and 2.

In our implementations, we calculate the highest quadratic function passing through the given end points and having the required slope at one of the end points. It can be shown that this function is entirely contained in the indicated triangle and is convex.

It is clear that the fundamental property exploited in calculating the above bounds is the convexity of the optimal value function. Thus, the use of this procedure is not limited to problems of the form $CR(\varepsilon)$, but can be applied to any parametric problem that has a convex optimal value. In fact, it is obvious that this technique is also applicable to parametric programs with concave optimal value functions. The only minor alteration is that Step 3 of the algorithm will provide the lower bound, while Step 4 will yield the upper bound.

The relevance of the application of the above technique to the problem under consideration in the present study is that the posynomial programs, under the usual exponential transformation, becomes a convex problem. Since the transformation does not affect the value of the problem functions, the above technique may obviously be applied to calculate parametric bounds on the optimal value function of GP problems with respect to right-hand side perturbations. The convexity of the optimal value of the posynomial GP as a function of the right-hand side perturbations follows immediately from the exponential transformation and a well-known result. Furthermore, it follows that the optimal value function of the primal GP (posynomial) problem (formulated as a minimization problem) is a monotone nondecreasing <u>concave</u> function of the coefficients appearing in the GP primal objective function. This follows from the linearity of the objective function as a function of these coefficients and the fact that $f^*(\varepsilon) = \min_R f(x,\varepsilon)$ is concave if $f(x,\varepsilon)$ is concave in ε and R does not depend on ε. Concavity with respect to each ε_i was noted by Dembo [2], who did not make use of the general result indicated but calculated the second derivatives of the optimal value function. In Section 4, using these results, we are

able to apply the given technique to derive piecewise linear parametric bounds on the optimal value function of the model under consideration as a function of the model parameters.

3. The Model

As mentioned earlier, it is not the purpose of this study to justify the underlying theory and formulation of the resulting GP problem. We list the problem variables and parameters, followed by a concise statement of the model objective function and constraints. Readers interested in details regarding the derivation of the model may refer to [3], [4] and [15].

The model under consideration requires the minimization of the total cost involved in the steam turbine power plant as a function of the turbine exhaust annulus and the condenser system design, subject to a multitude of engineering and thermodynamic constraints.

Model variables

The variables involved in the model and their description, extracted from [3], are as follows:

A_{an} : total turbine exhaust annulus area, sq ft

T_s : saturation temperature of steam, ^{o}F

N : number of tubes in condenser

D_o : outside tube diameter, ft

D_i : inside tube diameter, ft

L : condenser tube length, ft

q : rate of heat transfer, Btu/hr

Q_a : condenser flow, lb/hr

f_M : moisture correction factor at expansion line end point

Δh : exhaust loss, Btu/lb

T_h : hot water temperature, ^oF

Model parameters

There are numerous parameters involved in the derivations of the models developed in [15] by Wiebking. Here we will only state those parameters which are explicitly involved in the formulation of the model considered here:

$a_i (i=1,6)$: regression coefficient involved in derivation of emperical formulas for the model

B : pressure drop factor

b_i (i=1,5 and i=61, 62) : regression coefficients involved in derivation of emperical formulas for the model

b_{an} : exponent in annulus area cost relation

c_{an} : coefficient in annulus area cost relation

c_E : cost of electricity, \$/(kW-hr)

c_p : specific heat, Btu/(lb-^oF)

c_t : unit cost of condenser surface, \$/sq ft

$D_{o,min}$: minimum allowable outside tube diameter, ft

$f_{M,R}$: moisture correction factor at rated average conditions

g : gravitational constant, ft/sec^2

$h_{2,R}$: expansion line, end point, at rated average conditions, Btu/lb

k : thermal conductivity, Btu/ft-hr-^oF)

ℓ : tube wall thickness, ft

Pr : Prandtl number

P_c : rate of depreciation per year

$Q_{a,R}$: condenser flow at rated average conditions, lb/hr

T_c : cold water temperature, ^oF

t : load factor - effective operating time, hours/year

u_{min} : minimum tube velocity, ft/sec

V_L : lower bound on annulus velocity of steam, ft/sec

V_U : upper bound on annulus velocity of steam, ft/sec

W : water flow rate, lb/hr

η_G : generator efficiency

η_P : pump efficiency

λ : latent heat of evaporation, Btu/lb

μ : viscosity, lb/(ft-hr)

ρ : density, lb/cu ft

τ_{max} : allowable last stage bucket loading, lb/(hr-sq ft)

ΔT_{max}: maximum allowable temperature increase of cooling water, ^oF

f : outside film, a subscript

The objective function g_0

The model minimizes g_0 , the total annual cost involved in the steam turbine power plant as a function of the exhaust annulus and the condenser system design, and consists of two parts: fixed costs and operating costs. Denoting the fixed and operating costs of the turbine by $C_{F,T}$ and $C_{O,T}$ and those of the condenser by $C_{F,C}$ and $C_{O,C}$, respectively, the total annual cost to be minimized is

$$g_0 = (C_{F,T} + C_{O,T}) + (C_{F,C} + C_{O,C}) .$$

The formulas derived in [3] and [4] for the above cost components, as a function of the design variables and parameters, are as follows:

$$C_{F,T} = c_1 \; A_{an}^{\;b_{an}} \qquad \$/\text{year}$$

$$C_{O,T} = c_2 \; (q-q_R) + \text{fuel cost} \qquad \$/\text{year}$$

$$C_{F,C} = c_3 \; D_o LN \qquad \$/\text{year}$$

$$C_{O,C} = c_4 \; L/(D_i^{4.8} * N^{1.8}) \qquad \$/\text{year}$$

where

$$c_1 = c_{an} \; P_c$$

$$c_2 = c_E t \; \eta_G/3412.75$$

$$c_3 = \pi c_t P_c$$

$$c_4 = (32)(.046) \; C_E Bt \; W^{2.8} \; (\mu/4)^{.2}/(g\eta_p \; \rho^2 \; \pi^{1.8})$$

and q_R is the rate of heat transfer at the rated average conditions in Btu/hr.

Notice that the operation cost of the turbine $C_{O,T}$ consists of the replacement energy cost and the fuel cost. In the formulation of the model, it is assumed that the turbine generator runs at full load, implying that the cost of fuel is constant. Also, in replacement cost terms, the quantity $c_2 q_R$ is constant. Thus, the fuel cost and $c_2 q_R$ are deleted from consideration in the optimation model.

The Constraints

(i) constraint on condenser rejected heat

$$g_1: \; c_5 \frac{Q_a}{q} + c_6 \frac{Q_a T_s}{q} + c_7 \frac{Q_a f_M \, \Delta h'}{q} \le 1$$

where

$$c_5 = b_{2,R} + a_1 f_{M,R} + .87\ a_4 a_6 - a_2$$

$$c_6 = b_1 f_{M,R} + .87\ b_4 a_6 - b_2$$

$$c_7 = 1$$

$$\Delta h' = \Delta h - a_6$$

(ii) Moisture correction factor constraint

$$g_2 : c_8 \frac{1}{f_M} + c_9 \frac{T_s}{f_M} \leq 1$$

where

$$c_8 = .87\ a_4$$

$$c_9 = .87\ b_4$$

(iii) Exhaust loss constraint

$$g_3 : c_{10} \frac{T_s^{2b_5}}{A_{an}^2\ \Delta h'} + c_{11} \frac{A_{an}}{T_s^{b_5} \Delta h'} \leq 1$$

where

$$c_{10} = b_{61}$$

$$c_{11} = \frac{b_{62}}{a_5\ Q_{a,R}}$$

(iv) Heat exchanger mean temperature drop constraint

$$g_4 : c_{12} \frac{1}{T_s} + c_{13} \frac{q}{T_s} + c_{14} \frac{q^{4/3}}{T_s N^{7/6} D_o L^{4/3}} + c_{15} \frac{g\ D_i}{T_s L\ N^{.2}} \leq 1$$

where

$$c_{12} = T_c$$

$$c_{13} = 1/(2\ c_p\ W)$$

$$c_{14} = (.725\ \pi)^{-4/3} \left[\frac{\mu_f}{(2)(3600)^2\ k_f{}^3 \rho\ f^2 g\ \lambda}\right]^{1/3}$$

$$c_{15} = (.023\ \pi\ k\ Pr^{.4})^{-1} * \left(\frac{\pi\ \mu}{4W}\right)^{.8}$$

(v) Condenser flow constraint

$$g_5 :\ c_{16}\ \frac{1}{Q_a} + c_{17}\ T_s \leq 1$$

where

$$c_{16} = \frac{Q_{a,R}}{a_3} \quad \text{and}$$

$$c_{17} = -\ \frac{b_3}{a_3}$$

(vi) Heat exchanger tube diameter constraints

$$g_6 :\ c_{18}\ \frac{1}{D_o} + c_{19}\ \frac{D_i}{D_o} \leq 1 \quad \text{and}$$

$$g_7 :\ c_{20}\ \frac{1}{D_o} \leq 1$$

where

$$c_{18} = 2\ell$$

$$c_{19} = 1.$$

$$c_{20} = D_{o,min}$$

(vii) Annulus fluid velocity constraints

$$g_8 : c_{21} \frac{T_s^{b_5}}{A_{an}} \leq 1$$

$$g_9 : c_{22} \frac{A_{an}}{T_s^{b_5}} \leq 1$$

where

$$c_{21} = \frac{a_5 Q_{a,R}}{V_U}$$

$$c_{22} = \frac{V_L}{a_5 Q_{a,R}}$$

(viii) Last stage bucket loading constraint

$$g_{10} : c_{23} \frac{Q_a}{A_{an}} \leq 1$$

where

$$c_{23} = \frac{1}{\tau_{max}}$$

(ix) Heat exchanger tube water velocity constraint

$$g_{11} : c_{24} N D_i^2 \leq 1$$

where

$$c_{24} = \frac{3600 u_{min} \pi \rho}{4W}$$

(x) Thermal pollution constraint

$$g_{12} : c_{25} T_h \leq 1$$

$$c_{25} = \frac{1}{T_c + \Delta T_{max}}$$

Denoting the model variables A_{an} , T_s , N , D_o , D_i , L , q , Q_a , f_M , $(\Delta h \cdot t_w)$ and T_h by x_1 through x_{11} , respectively, the model is formulated as follows.

The GP problem

Minimize $_x$ $\quad g_0 \equiv c_1 x_1^{b_{an}} + c_2 x_7 + c_3 x_3 x_4 x_6 + c_4 x_6 x_5^{-4.8} x_3^{-1.8}$

subject to: $\quad g_1 \equiv c_5 x_8 x_7^{-1} + c_6 x_8 x_2 x_7^{-1} + c_7 x_8 x_9 x_{10} x_7^{-1} \leq 1$

$$g_2 \equiv c_8 x_9^{-1} + c_9 x_2 x_9^{-1} \leq 1$$

$$g_3 \equiv c_{10} x_2^{2b_5} x_1^{-2} x_{10}^{-1} + c_{11} x_1 x_2^{-b_5} x_{10}^{-1} \leq 1$$

$$g_4 \equiv c_{12} x_2^{-1} + c_{13} x_7 x_2^{-1} + c_{14} x_7^{4/3} x_2^{-1} x_3^{-7/6} x_4^{-1} x_6^{-4/3} + c_{15} x_7 x_5^{.8} x_2^{-1} x_6^{-1} x_3^{-.2} \leq 1$$

$$g_5 \equiv c_{16} x_8^{-1} + c_{17} x_2 \leq 1$$

$$g_6 \equiv c_{18} x_4^{-1} + c_{19} x_5 x_4^{-1} \leq 1$$

$$g_7 \equiv c_{20} x_4^{-1} \leq 1$$

$$g_8 \equiv c_{21} x_2^{b_5} x_1^{-1} \leq 1$$

$$g_9 \equiv c_{22} x_1 x_2^{-b_5} \leq 1$$

$$g_{10} \equiv c_{23}\, x_8\, x_1^{-1} \leq 1$$

$$g_{11} \equiv c_{24}\, x_3\, x_5^{2} \leq 1$$

$$g_{12} \equiv c_{25}\, x_{11} \leq 1$$

4. Model Solution

Ecker and Wiebking [3], using a data base drawn from [15], have solved this model. In [3], they conclude from a preliminary analysis that constraints $g_8 - g_{12}$, with the given data base, are not binding and thus ignore these constraints in determining a solution to the GP problem. Henceforward, the problem with $g_8 - g_{12}$ deleted will be the focus of our attention, and will be called the "reduced problem."

The values of coefficients c_1 through c_{20} in [3] are given as follows:

$c_1 = 2541.0$	$c_6 = .23972$	$c_{11} = 6.2139 * 10^{-8}$	$c_{16} = 2.447 * 10^{6}$
$c_2 = .012293$	$c_7 = 1.0$	$c_{12} = 50.0$	$c_{17} = 1.0289 * 10^{-3}$
$c_3 = 2.4190$	$c_8 = .62004$	$c_{13} = 4.7394 * 10^{-9}$	$c_{18} = 8.1667 * 10^{-3}$
$c_4 = 77171.0$	$c_9 = 1.1072 * 10^{-3}$	$c_{14} = 7.3124 * 10^{-1}$	$c_{19} = 1.0$
$c_5 = 888.76$	$c_{10} = 2.1872 * 10^{16}$	$c_{15} = 1.2577 * 10^{-5}$	$c_{20} = .083333$

Using the above coefficients, they obtain the solution of the reduced model by means of a modified concave simplex algorithm that solves the dual geometric program. The solution, obtained after 300 seconds of CPU time on an IBM system 360/50 computer, is listed below.

Dual objective function: $32.3728 \cdot 10^{6}$

Primal objective function: $32.6356 \cdot 10^6$

Primal variables:

$q = 2.5503 \cdot 10^9$	$T_s = 85.455$
$A_{an} = 409.16$	$N = 13,662$
$f_M = 0.71520$	$D_i = 0.082461$
$Q_a = 2.7010 \cdot 10^6$	$D_o = 0.091950$
$\Delta h' = 46.882$	$L = 70.606$

To solve the reduced GP problem in this study, the convex equivalent of the (reduced) model was formulated via the usual exponential transformation, $x_i = e^{-t_i}$, $i=1$, 10, and the same coefficients c_i used by Ecker and Wiebking and reported in [3] were utilized. To prevent over-and under-flow in the process of obtaining the numerical solution, the following constraints (which proved to be nonbinding at the optimal solution) were added to the model:

$$-170 \leq t_6 - 4.8\, t_5 - 1.8\, t_3 \leq 170$$

$$-30 \leq t_i \leq 30 \quad i=1,10$$

The resulting problem was solved by SENSUMT, on The George Washington University IBM 3031. The solution, obtained in 57 seconds of CPU time, is listed below.

Dual objective function $32.37324 * 10^6$

Primal objective function $32.37325 * 10^6$

$x_1 = A_{an} = 408.88$	$x_6 = L = 58.46230$
$x_2 = T_s = 85.452$	$x_7 = q = 2.52929 * 10^9$
$x_3 = N = 18979.5$	$x_8 = Q_a = 2.68298 * 10^6$
$x_4 = D_o = .07902$	$x_9 = f_M = .71465$
$x_5 = D_i = .07085$	$x_{10} = \Delta h' = 46.88288$

The significantly greater accuracy of this solution, based on the gap between the values of the dual and primal objective functions, compared to that obtained in [3], is apparent from the given data. The two solution vectors, although considerably different in some components, are of the same order of magnitude. As noted, the CPU time required by SENSUMT to obtain a relatively very accurate solution of the given model was about 1/5 of the CPU time spent by the algorithm in [3] to obtain a rather crude estimate of a solution. Of course, since the IBM 3031, is faster than the IBM 360/50 and since we do not know the initial point used in [3] to solve the problem, it is not possible to make a definitive comparison of the relative efficiency of the subject codes.

In the next section, we analyze the reduced model for sensitivity information and optimal value parametric bounds.

5. Optimal Value Function Sensitivities and Bounds

5.1 Optimal value function sensitivities

To conduct sensitivity and bound calculations with respect to the various parameters involved in the model, the coefficient c_i (i=1, 20) must be expressed as a function of the model parameters, rather than being considered independent and assigned fixed values (as was done in defining the data of the problem solved in Section 4). To accomplish this, the formulas taken from [4] and given in Section 3 for c_i (i=1, 20) were coded. Using the values for the various problem parameters, given in [3] and [15] and shown in Table 1, the following values were calculated for the components of the parameter - dependent coefficient vector $c=(c_1,\ldots,c_{20})$.

$c_1 = 1270.5$	$c_6 = .2397342$	$c_{11} = 6.213914 * 10^{-8}$	$c_{16} = 2.477010 * 10^{6}$
$c_2 = .01229334$	$c_7 = 1.0$	$c_{12} = 50.0$	$c_{17} = 1.028863 * 10^{-3}$
$c_3 = 2.419023$	$c_8 = .6200489$	$c_{13} = 4.739336 * 10^{-9}$	$c_{18} = 8.166663 * 10^{-3}$
$c_4 = 77169.87$	$c_9 = 1.107162 * 10^{-3}$	$c_{14} = 7.312399 * 10^{-6}$	$c_{19} = 1.0$
$c_5 = 888.7634$	$c_{10} = 2.18723 * 10^{18}$	$c_{15} = 1.2577 * 10^{-5}$	$c_{20} = .08333296$

Table 1

Problem Data

Symbol	Level	Symbol	Level
a_1	-159.9	K_f	.3630
a_2	-32.08	ℓ	.06408
a_3	1.1087	Pr	8.277
a_4	0.7127	P_c	.154
a_5	92340.	$Q_{a,R}$	$2.713 * 10^6$
a_6	-27.444	T_c	50.0
B	1.2	t	6337.86
b_1	1.7599	u_{min}	5.0
b_2	1.0	V_L	250
b_3	$-1.1407 * 10^{-3}$	V_U	1400
b_4	$1.2726 * 10^{-3}$	W	$1.055 * 10^8$
b_5	-3.0122	η_G	.988
b_{61}	$3.4851 * 10^{-5}$	η_P	.8
b_{62}	$1.5567 * 10^4$	λ	1037.2
b_{an}	1.		
c_{an}	8250.	μ	2.808
c_E	.00067	μ_f	1.6402
c_p	1.0	ρ	61.63
c_t	5.0	ρ_f	61.38
$D_{o,min}$	.083333	τ_{max}	15000
$f_{M,R}$	.7217	ΔT_{max}	30.0
g	32.174		
$h_{2,R}$	989.1		
k	.3393		

Except for c_1 , the components of c are in close agreement with those used in [3] and given in Section 4. (We note that in [15], the value of c_1 conforms exactly to the above value, which is 1/2 of the value reported in [3]. This suggests that the value of c_1 derived here is correct.) The results reported in the remainder of the paper are consequently based on these components of c .

Table 2 depicts the sensitivity of the optimal total cost with respect to numerous parameters of the model. As shown in this table, we do not conduct sensitivity analysis with respect to all of the model data (parameters) listed in Table 1. This is because some data cannot be meaningfully altered. For example, the gravitational constant g is assumed known and fixed. The specific heat c_p , the viscosity μ , etc., are internal properties of the fluid and cannot freely be altered.

As elaborated in our earlier papers, [8], [9], the sensitivity measure $\partial f^*(\epsilon)/\partial\epsilon_i$ corresponds to the rate of change of the optimal value function $f^*(\epsilon)$ with respect to ϵ_i , at a given value of ϵ . It would seem that the larger the absolute value of $\partial f^*(\epsilon)/\partial\epsilon_i$, the greater the sensitivity of $f^*(\epsilon)$ with respect to ϵ_i . However, it would be erroneous to conclude this and misleading to use the information without further analysis, as a guideline for making and implementing appropriate decisions. This is, in part, because this rate of change is local and does not necessarily reflect the change that can result from finite data changes. Nor does this measure reflect the likelihood of a given finite change. For such reasons, it is imperative to calculate some measures of stability in the large, to estimate or at least contain the effects of finite data changes on the optimal solution. Moreover, sensitivities are often interpreted as the "change in $f^*(\epsilon)$ per unit change in ϵ_i ". Such interpretations may also prove to be misleading because they tacitly suggest that a unit change is meaningful and that it might be realized in a given parameter ϵ_i . However, a one unit change for one parameter may be too large to be practically feasible, while for

Table 2

Optimal Value Function Sensitivity Results

(1) Parameter No. i	(2) Parameter ε_i	(3) Parameter Value $\bar{\varepsilon}_i$	(4) Sensitivity $\partial f^*(\bar{\varepsilon})/\partial\varepsilon_i$	(5) Scaled Sensitivity $\lvert .01\bar{\varepsilon}_i\rvert (\partial f^*(\bar{\varepsilon})/\partial\varepsilon_i)$	(6) Rank
1	a_1	-159.9	22.577.27	36,101.05	8
2	a_2	- 32.08	-31,283.56	-10,035.77	15
3	† a_3	1.1087	$-.3294094 * 10^8$	-365,216.2	3
4	a_4	.7127	341,840.9	2,436.30	24
5	a_5	92340	8.288348	7,653.46	17
6	a_6	-27.444	22,112.94	6,068.67	19
7	B	1.2	33,954.16	407.45	26
8	b_1	1.7599	1,770.420	31,157.62	9
9	b_2	1.0	-2,453,125.	-24,531.25	12
10	b_3	$-1.1407 * 10^{-3}$	$-.2583156 * 10^{10}$	-29,466.06	11
11	b_4	$1.2716 * 10^{-3}$	$.2680632 * 10^8$	341.14	27
12	b_5	-3.0122	3,316,445.	99,897.95	6
13	b_{61}	$3.4851 * 10^{-5}$	$.1728371 * 10^{11}$	6,023.55	20
14	b_{62}	$1.5567 * 10^4$	28.58427	4,449.71	22
15	b_{an}	1.	4,710,256.	47,102.56	7
16	† c_{an}	8250.	80.40698	7,376.07	18
17	c_E	.00067	$.4588196 * 10^{10}$	30,740.91	10
18	† c_t	5.0	55,620.45	2,781.02	23
19	ℓ	.06408	3,797,937.	154.96	28

continued

Table 2 (continued)

Optimal Value Function Sensitivity Results

(1) Parameter No. i	(2) Parameter ϵ_i	(3) Parameter Value $\bar{\epsilon}_i$	(4) Sensitivity $\partial f^*(\bar{\epsilon})/\partial\epsilon_i$	(5) Scaled Sensitivity $\lvert .01\bar{\epsilon}_i \rvert (\partial f^*(\bar{\epsilon})/\partial\epsilon_i)$	(6) Rank
20	P_c	.154	6,609,746.	10,179.01	14
21	T_c	50.0	17,350.95	8,675.47	16
22	† t	6337.86	4,850.329	307,407.06	4
23	W	$1.055 * 10^8$	-.0175986	18,566.52	13
24	† η_G	.988	$.3107304 * 10^8$	307,001.64	5
25	P_{mi} †$(=\frac{1}{\eta_P})$	$1.25(=\frac{1}{.8})$	32,596.29	407.453	25
26	$Q_{a,R}$	$2.713 * 10^6$	12.97611	352,041.86	1
27	$h_{2,R}$	989.1	31,283.35	309,423.61	2
28	$f_{M,R}$	.7217	-684,874.7	- 4,942.74	21

† Candidate for parametric bound calculation on $f^*(\epsilon)$.

another it may be too small to be numerically significant. Parameters η_G , the turbine efficiency, and W , the water flow rate, in Table 2 provide good examples of both instances.

With these points in mind, and assuming that the sensitivities are acceptable in a small (finite) neighborhood of the problem data, we prefer to base our judgments on "scaled" sensitivities such as those given in Column (5) of Table 2, which correspond to the estimated changes resulting from a one percent change in a given parameter value. Column 6 of this table ranks the above scaled sensitivity measures, in decreasing order of their absolute values. The importance of providing the kind of information depicted in Table 2 for real world systems seems obvious and cannot be overstated. For example, it provides guidelines for setting priorities on the relative importance of the parameters, and suggests actions that can be taken to obtain optimum marginal improvements in the optimum performance.

5.2 Optimal value function bounds

Here, we apply the bound calculation technique discussed in Section 2, to derive piecewise linear parametric upper and lower bounds on the optimal total cost of the steam turbine power plant under study.

The parameters marked by (†) in Table 2 are retabulated in Table 3 and are selected for bound calculation. Notice that parameter a_3 is, in fact, a right-hand side parameter of constraint g_5 . Recall that

$$g_5 = c_{16} x_8^{-1} + c_{17} x_2 \leq 1$$

where

$$c_{16} = \frac{Q_{a,R}}{a_3} \quad \text{and} \quad c_{17} = -\frac{b_3}{a_3}$$

Thus g_5 can be written as

$$Q_{a,R} x_8^{-1} - b_3 x_2 \leq a_3 .$$

Thus, optimal value function of the model under study is a convex non-increasing function of a_3 .

Table 3

Parameters Selected For Bound Calculation

Parameter No.	Parameter Name	Parameter Value	Perturbation
3	a_3	1.1087	.2527
16	c_{an}	8250.	-2062.5
18	c_t	5.0	-1.25
22	t	6337.86	-1584.465
24	η_C	.988	-.247
25	$P_{mi}(= \frac{1}{\eta_p})$	1.25 $(= \frac{1}{.8})$	-.125

Also, parameters c_{an} , c_t , t and η_G, taken one at a time, appear in the objective function coefficients linearly and do not appear in the constraints. Thus, the optimal annual cost of the plant under consideration is a concave nondecreasing function of each of these parameters. This means that our bound calculation procedure can readily be applied to these parameters. Notice that parameter η_P (pump efficiency) does not appear linearly in the objective function coefficients. For this reason, we replace $\frac{1}{\eta_P}$ by P_{mi} . This enables us to calculate optimal value bounds relative to changes in P_{mi} and, hence, relative to changes in η_P .

The last column in Table 3 shows the magnitude of the perturbations allowed in the respective parameter value, each taken in the direction that decreases the optimal value function, as determined by the sensitivity information in Table 2. In the following, we list the resulting parametric bounds on the optimal value over the given range of the parameter, obtained by our latest version of the computer programming code SENSUMT [10].

- Bounds w.r.t. a_3 , $1.1987 \leq a_3 \leq 1.3614$

Upper bound $\bar{F} = -.1792469 * 10^8\, a_3 + .5163132 * 10^8$

Lower bound $\underline{F} = \text{Max}\,\{-.3294094 * 10^8\, a_3 + .6827984 * 10^8,$

$-.6760806 * 10^7\, a_3 + .3643282 * 10^8\}$

Figure 2 depicts the resulting computer output for this and supplemental bound calculation for a_3 .

- Bounds w.r.t. c_{an} , $6187.5 \leq c_{an} \leq 8250$.

Optimal Value Function Bounds When Par(3) is Perturbed

Point 1 (Unperturbed Solution) :
Par(3) = 0.1108700D 01 F(Par(3)) = 0.3175823D 08

Point 2 (Perturbed Solution) :
Par(3)) = 0.1361400D 01 F(Par(3)) = 0.2722866D 08

Line passing through Points 1 and 2 and overestimating f*

F = -0.1792469D 08 * Par(3) + 0.5163132D 08

Line underestimating f* at Point 1

F = -0.3294094D 08 * Par(3) + 0.6827984D 08

Line underestimating f* at Point 2

F = -0.6760806D 07 * Par(3) + 0.3643282D 08

Quadratic Estimation of f* through Points 1 and 2

F = 0.4417841D 08 * Par(3)**2 - 0.1270497D 09 * Par(3) + 0.1183135D 09

f* bound evaluation at ten equidistant points between Points 1 and 2

Par(3)	Lower Bound	Upper Bound	Quad Estimate
1.109	0.3175823D 08	0.3175823D 08	0.3175823D 08
1.134	0.3092581D 08	0.3130527D 08	0.3105137D 08
1.159	0.3009339D 08	0.3085231D 08	0.3040093D 08
1.185	0.2926097D 08	0.3039935D 08	0.2980692D 08
1.210	0.2842856D 08	0.2994640D 08	0.2926933D 08
1.235	0.2808288D 08	0.2949344D 08	0.2878816D 08
1.260	0.2791204D 08	0.2904048D 08	0.2836342D 08
1.286	0.2774119D 08	0.2858753D 08	0.2799509D 08
1.311	0.2757035D 08	0.2813457D 08	0.2768319D 08
1.336	0.2739950D 08	0.2768161D 08	0.2742771D 08
1.361	0.2722866D 08	0.2722866D 08	0.2722866D 08

Figure 2. Parametric bounds on $f^*(a_3)$

Upper bound $\bar{F} = \min\ \{89.40698\ c_{an} + .3102062 * 10^8\ ,$

$100.5829\ c_{an} + .3094000 * 10^8\}$

Lower bound $\underline{F} = 94.96585\ c_{an} + .3097476 * 10^8$

. Bounds w.r.t. c_t, $3.75 \leq c_t \leq 5.0$

Upper bound $\bar{F} = \min\ \{55620.45\ c_t + .3148012 * 10^8\ ,$

$60878.7\ c_t + .3145717 * 10^8\}$

Lower bound $\underline{F} = 58209.10\ c_t + .3146718 * 10^8$

Figure 3 depicts the resulting computer output for bound calculation with respect to c_t .

. Bounds w.r.t. t, $4753.395 \leq t \leq 6337.86$

Upper bound $\bar{F} = \min\ \{4850.329\ t + .1017524 * 10^7\ ,$

$4860.131\ t + .8543850 * 10^6\}$

Lower bound $\underline{F} = 4863.883\ t + .9316157 * 10^6$

. Bounds w.r.t η_G , $.741 \leq \eta_G \leq .988$

Upper bound $\bar{F} = \min\ \{.3107304 * 10^8\ \eta_G + .1058064 * 10^7\ ,$

$.3126812 * 10^8\ \eta_G + .8912627 * 10^7\}$

Lower bound $\underline{F} = .3116310 * 10^8\ \eta_G + .9690873 * 10^6$

Optimal Value Function Bounds When Par(18) is Perturbed

Point 1 (Unperturbed Solution) :
Par(18) = 0.5000000D 01 F(Par(18)) = 0.3175823D 08

Point 2 (Perturbed Solution) :
Par(18)) = 0.3750000D 01 F(Par(18)) = 0.3168546D 08

Line passing through Points 1 and 2 and underestimating f*

F = 0.5820910D 05 * Par(18) + 0.3146718D 08

Line overestimating f* at Point 1

F = 0.5562045D 05 * Par(18) + 0.3148012D 08

Line overestimating f* at Point 2

F = 0.6087870D 05 * Par(18) + 0.3145717D 08

Quadratic estimation of f* through Points 1 and 2

F = -0.2070913D 04 * Par(18)**2 + 0.7632959D 05 * Par(18) + 0.3142835D 08

f* bound evaluation at ten equidistant points between Points 1 and 2

Par(18)	Lower Bound	Upper Bound	Quad Estimate
5.000	0.3175823D 08	0.3175823D 08	0.3175823D 08
4.875	0.3175095D 08	0.3175127D 08	0.3175124D 08
4.750	0.3174367D 08	0.3174432D 08	0.3174419D 08
4.625	0.3173640D 08	0.3173737D 08	0.3173708D 08
4.500	0.3172912D 08	0.3173042D 08	0.3172990D 08
4.375	0.3172184D 08	0.3172346D 08	0.3172265D 08
4.250	0.3171457D 08	0.3171590D 08	0.3171535D 08
4.125	0.3170729D 08	0.3170829D 08	0.3170797D 08
4.000	0.3170002D 08	0.3170068D 08	0.3170053D 08
3.875	0.3169274D 08	0.3169307D 08	0.3169303D 08
3.750	0.3168546D 08	0.3168546D 08	0.3168546D 08

Figure 3. Parametric bounds on $f^*(c_t)$

. Bounds on $P_{mi} = \frac{1}{\eta_p}$, $1.125 \leq \frac{1}{\eta_p} \leq 1.25$ $(.8 \leq \eta_p \leq .889)$

Upper bound $\bar{F} = \min \{.3259629 * 10^5 \frac{1}{\eta_p} + .3171748 * 10^8 ,$

$.3536232 * 10^5 \frac{1}{\eta_p} + .3171425 * 10^3\}$

Lower bound $\underline{F} = .3355953 * 10^5 \frac{1}{\eta_p} + .3171628 * 10^8$

These bounds are piecewise linear as a function of P_{mi} , hence they are nonlinear as a function of η_p .

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